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HEAT TRANSFER AND HEAT CONDUCTION IN TECHNOLOGICAL PROCESSES

NUMERICAL SIMULATION OF HIGH-TEMPERATURE THERMAL PROCESSES IN CYLINDRICAL FURNACES

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A mathematical model of the high-temperature process of heat transfer in a cylindrical furnace has been constructed. Based on this model, investigations and assessment of the contribution made by longitudinal radiation to the process of radiant heat transfer in the cylindrical furnace were carried out. It has been established that the assumption on the exclusively radial propagation of a radiant flux may lead to considerable errors. It is shown that the zone method in conjunction with the method of Monte Carlo statistical tests is efficient for solving the problem of radiative heat transfer.

In the chemical and metallurgical industries and in the building materials industry wide use is made of rotating roasting furnaces. They have a cylindrical shape, a length from 50 to 150 m, and a diameter from 3 to 5 m. In solving the problems of the design and optimization of such units mathematical simulation plays an important role.

Mathematical models of rotating furnaces are available at the present time [1–3]. However, they admit only radial propagation of thermal radiation and ignore the longitudinal one. It should be taken into consideration here that the high temperatures (above 1500^{o}C) inside a furnace and the considerable diameter of the latter make the influence of the longitudinal radiation appreciable.

The aim of the present work is the mathematical simulation and investigation of the thermal processes occurring inside a furnace with account for the longitudinal radiation. The goal of the present investigation is the assessment of the influence of the longitudinal radiation; therefore the following simplifications were made when constructing the mathematical model:

1) the inner space of the furnace represents a cylindrical volume without a material;
2) there are no heat losses through the lining;
3) natural gas is burnt inside the furnace.

Model of Burning out of a Fuel Flame. In calculating the curves of the burning out of a fuel flame and air inflow into the zone of combustion along the furnace length, the length of the flame is adopted in [4] to be the distance from the nozzle outlet to the section where incomplete combustion of the fuel amounts to 2%. The length of the flame l_f is calculated on the basis of the structural characteristics of the fuel combustion system [5]. The integral inflow of air into the flame $\beta(l)$ and the degree of the burning out $\chi(l)$ at the given length of the flame are calculated from the exponential relations [4]

$$\beta(l) = \vartheta(1 - \exp(-abl)) , \quad (1)$$

$$\chi(l) = 1 - \exp(-al^2) . \quad (2)$$

The coefficient a is determined from the condition $\chi(l) = 0.98$ at $l = l_f$. The coefficient b can be found by substituting the values $\beta = 1$ and $\chi = 0.85$ into Eqs. (1) and (2). The value of l at which the inflow value $\beta(l)$ is attained is called

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the inflow path length l_{inf}. The air excess coefficient ϑ is the ratio between the amount of air drawn over the path to the flame section considered and the theoretically needed amount of air.

Having the dependence for $\chi(l)$, we may determine the heat release from the burning fuel along the flame length. To calculate the heat transfer rate it is necessary to know the radiative properties of the flame. In [5] it was proposed to relate the coefficient of absorption of sooty particles of the flame $K_s(l)$ to the air inflow coefficient $\beta(l)$ and thus to describe the change in $K_s(l)$ along the flame length. For a gas flame

$$K_s(l) = 0.425 \cdot (1.0 - \beta(l)) . \quad (3)$$

In [6], a formula is suggested to calculate the integral emissivity $\varepsilon_g(l)$ of a natural gas which was obtained by approximating experimental data in the ranges of temperatures and pressures typical of industrial furnaces:

$$\varepsilon_g(l) = 2 \cdot 10^{-12} \left(775 \sqrt[3]{P_\alpha(l)} + 4845 - 8.27\,(5.9 - S_{ef})^3 - T_g(l) \right)^3 , \quad (4)$$

$$S_{ef} = 0.9 \cdot \frac{4V_g}{F_{lin}} . \quad (5)$$

For a gaseous fuel the relationship between the total content of steam and carbon dioxide P_α with the degree of burning-out $\chi(l)$ and inflow $\beta(l)$ is defined in [6] as

$$P_\alpha = \frac{P_{th} \chi(l)\, V_{th}}{\chi(l)\, V_{th} + (1 - \chi(l)) + (\beta(l) - \chi(l))\, L_{th}} , \quad (6)$$

$$P_{th} = \frac{V_{CO_2} + V_{H_2O}}{V_{th}} \cdot 100\% . \quad (7)$$

The composition and volume of combustion products V_{th}, L_{th}, V_{CO_2}, and V_{H_2O} are determined by the well-known technique [7, 8]. The relationship between the emissivity $\varepsilon_g(l)$ and the coefficient of absorption of combustion products $K_g(l)$ has the form [9]

$$\varepsilon_g(l) = 1 - \exp(-K_g(l)\, S_{ef}) . \quad (8)$$

If we neglect the dustiness of the gas, the coefficient of absorption $K_\Sigma(l)$ of the gas flow is equal to the sum of the coefficients of absorption of the sooty particles in the flame $K_s(l)$ and of the gaseous products of fuel combustion $K_g(l)$:

$$K_\Sigma(l) = K_s(l) + K_g(l) . \quad (9)$$

Heat Transfer Processes. We will divide the furnace along its length into N identical sections (Fig. 1) [10]. We will consider a section as an object with lumped parameters. We assume that the space inside the furnace is filled with an absorbing gray medium, it is bounded by gray surfaces, and that the thermal radiation considered is diffuse. The thermal balance for the volumetric zone of the gas medium of the lth section has the form

$$Q_{g,rl} + \Delta Q_{gl} - Q_{lin\,l} + Q_{com\,l} = 0 , \quad l = 1, 2, ..., N . \quad (10)$$

The thermal balance for the lining of the lth section is

$$Q_{lin,rl} + Q_{lin\,l} = 0 , \quad l = 1, 2, ..., N . \quad (11)$$

The quantity ΔQ_{gl} that characterizes heat transfer with the moving gas is

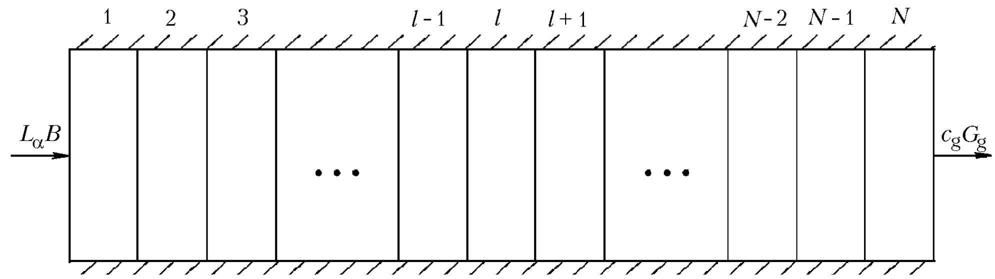

Fig. 1. Representation of a furnace as a sequence of sections.

$$\Delta Q_{gl} = c_{gl-1} G_{gl-1} t_{gl-1} - c_{gl} G_{gl} t_{gl} . \tag{12}$$

The quantity of heat transferred from the gas to the lining is defined as

$$Q_{\text{lin}l} = \alpha F_{\text{lin}} (T_{gl} - T_{\text{lin}l}) , \quad F_{\text{lin}} = 2\pi r \Delta l . \tag{13}$$

Based on investigation of convective heat transfer during motion of air in a tube, in [11] the generalized dependence Nu = $0.018 \cdot \text{Re}^{0.80}$ was derived. Substituting the values of Nu and Re (Nu = $\alpha D/\lambda$, Re = $\rho w/\nu$) into this relation we obtain the following equation for α:

$$\alpha = 0.018 \cdot \frac{\lambda}{\nu D^{0.20}} w^{0.80} . \tag{14}$$

We will calculate the curve of the burning-out of a fuel flame by relation (2). The quantity of heat released in the lth section on combustion of fuel is given by

$$Q_{\text{com}l} = q (\chi_l - \chi_{l-1}) B . \tag{15}$$

The quantity and composition of combustion products L_α, V_α, the combustion heat q at known percentage composition of fuel, and the air excess coefficient ϑ are determined on the basis of the well-known formulas given in [7, 8]. The volumetric rate of gas flow at the exit from the lth section is determined from the obvious relation

$$G_{gl} = L_\alpha (1 - \chi_l) B + V_\alpha \chi_l B .$$

We will assume that the inner space of the furnace consists of volumetric and surface zones. For convenience of representation and calculation we will number the zones. The total number of zones is $i = 2N$. The nomber of the volumetric zone of the gas flow corresponds to the nomber of the furnace section $i = l$ and changes from 1 to N. The nomber of the surface zone on the lining of the lth section is calculated as $i = l + N$ ($i = N + 1$, ..., $2N$).

The net radiation flux for the jth zone is

$$Q_{rj} = Q_{\text{inc}j} - Q_{\text{int}j} , \quad Q_{\text{inc}j} = \sum_{i=1}^{N} Q_{gij} + \sum_{i=N+1}^{2N} Q_{\text{lin}ij} , \tag{16}$$

$$Q_{gij} = 4 K_{\Sigma i} \sigma U f_{ij} T_{gl}^4 , \quad i = l , \quad Q_{\text{lin}ij} = \varepsilon_{\text{lin}} \sigma F_{\text{lin}} f_{ij} T_{\text{lin}l}^4 , \quad i = l + N . \tag{17}$$

The resolving angular coefficient of radiation f_{ij} determines the fraction of energy absorbed in the zone j from the energy radiated in the zone i, with account for multiple reflections from the bounding surfaces [5, 6]. Introducing the notation

$$A_{\rm g} = 4\sigma U \,, \quad A_{\rm lin} = \varepsilon_{\rm lin}\sigma F_{\rm lin} \,, \tag{18}$$

for the quantities of intrinsic thermal radiation of the jth zone, we write

$$Q_{{\rm g,int}j} = A_{\rm g} K_{\Sigma j} T_{{\rm g}j}^4 \,, \quad Q_{{\rm lin,int}j} = A_{\rm lin} T_{{\rm lin}j}^4 \,. \tag{19}$$

Subject to (10)–(19), we finally have N systems of two equations:

$$A_{\rm g} \sum_{i=1}^{N} K_{\Sigma i} f_{ij} T_{gl}^4 + A_{\rm lin} \sum_{i=N+1}^{2N} f_{ij} T_{{\rm lin}(i-N)}^4 - A_{\rm g} K_{\Sigma l} T_{gl}^4 + c_{gl-1} G_{gl-1} t_{gl-1}$$

$$- c_{gl} G_{gl} t_{gl} - \alpha F_{\rm lin} (T_{gl} - T_{{\rm lin}l}) + q\,(\chi_l - \chi_{l-1})\,B = 0 \,, \quad j = l \,, \quad l = 1, 2, ..., N \,; \tag{20}$$

$$A_{\rm g} \sum_{i=1}^{N} K_{\Sigma i} f_{ij} T_{gi}^4 + A_{\rm lin} \sum_{i=N+1}^{2N} f_{ij} T_{{\rm lin}(i-N)}^4 - A_{\rm lin} T_{{\rm lin}l}^4 + \alpha F_{\rm lin} (T_{gl} - T_{{\rm lin}l}) = 0 \,, \quad j = l + N \,, \quad l = 1, 2, ..., N \,. \tag{21}$$

The value of the longitudinal radiation incident on the jth zone of the lth section is estimated from the formula

$$Q_{{\rm long}j} = A_{\rm g} \sum_{\substack{i=1 \\ i\neq l}}^{N} K_{\Sigma i} f_{ij} T_{gi}^4 + A_{\rm lin} \sum_{\substack{i=N+1 \\ i\neq l+N}}^{2N} f_{ij} T_{{\rm lin}(i-N)}^4 \,. \tag{22}$$

The ratio of the longitudinal radiation incident on the zone j to the overall radiation flux incident on the zone j has the form

$$\Delta Q_{\rm long} = \frac{Q_{{\rm long}j}}{Q_{{\rm inc}j}} \cdot 100\% \,. \tag{23}$$

Determination of the Resolving Angular Radiation Coefficients. In order to solve the equations of the heat transfer model it is necessary to know the resolving angular coefficients f_{ij}. The technique of their determination in terms of generalized angular coefficients ψ_{ij} was given in [12]. The generalized angular coefficient of radiation ψ_{ij} determines the fraction of the radiant flux incident on the irradiated zone j of the entire radiant flux emitted by the ith zone. The passage from generalized angular coefficients ψ_{ij} to the resolving coefficients f_{ij} is made by solving systems of algebraic equations.

The fraction f_{ij} of the energy absorbed by the surface zone j from the energy emitted by zone i is determined by solving a system of linear algebraic equations:

$$f_{ij} = \psi_{ij}\varepsilon_j + \sum_{k=N+1}^{2N} R_k \psi_{ik} f_{ik} \,, \quad i = 1, 2, ..., 2N \,, \quad j = 1, 2, ..., 2N \,. \tag{24}$$

The fraction f_{ij} of the energy absorbed by the volumetric zone j is found by solving the system of equations

$$f_{ij} = \psi_{ij} + \sum_{k=N+1}^{2N} R_k \psi_{ik} f_{ik} \,, \quad i = 1, 2, ..., 2N \,, \quad j = 1, 2, ..., 2N \,. \tag{25}$$

Taking the lining to be an entirely opaque body, we may write

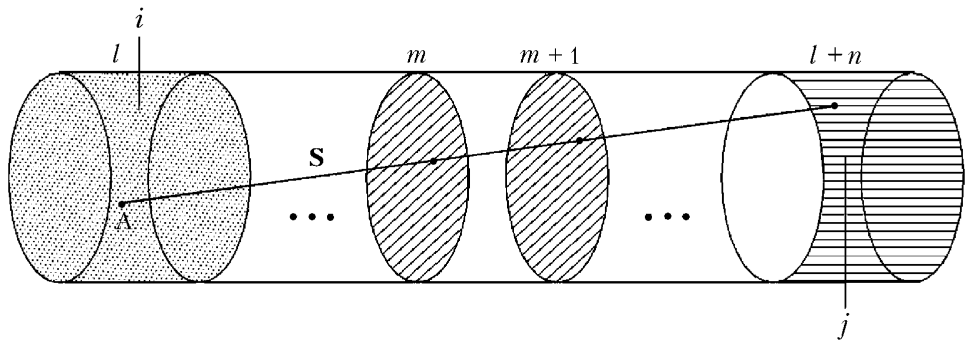

Fig. 2. Passage of radiation from the volumetric zone i of section l to the surface zone j of section $l + n$.

$$\varepsilon_j = 1 - R_j \,. \tag{26}$$

The physical meaning of Eqs. (24) and (25) is that the total arrival of radiant energy into the zone j from the emitting zone i consists of the energy due to direct emission of radiation from the zone i to the zone j and of the sum of radiant fluxes reflected into the zone j from each of the surface zones k as a result of the emission of radiation from the zone i.

The use of widely distributed techniques of determining generalized angular coefficients ψ_{ij} [6] based on computation of ternary integrals requires complex analytical transformations and is possible only in the most simple cases. The effective method of determining ψ_{ij} is the Monte Carlo method of statistical tests [5, 7, 13]. The present model employs a modification of this method, called in the literature the method of analytical averaging (or of statistical weights) [14–16]. The method of statistical tests makes it possible to restrict oneself to a series of calculations (numerical experiments): a particular choice from the entire set of random processes of radiation, transfer, and absorption of energy. A single test consists of the following stages:

1. Random sampling of an emitting point inside a radiating zone.
2. Random sampling of the direction of radiation.
3. Determination of the segments of the flow trajectory of selected direction up to the bounding surface.
4. Determination of the fraction of the radiant energy that reached the irradiated zone.

We will show the operation of the algorithm on a specific example. We consider (Fig. 2) the emitting zone i (the volumetric zone of section l) and the irradiated zone j (the surface of the lining of section l). It is necessary to determine the fraction absorbed in zone j from the radiant flux emitted by zone i. The total characteristic of the radiation field can be obtained if we track the flux emitted by all the elements in all directions.

Let a packet of photons, which is characterized by energy I_0, be emitted from the point A of the ith zone in the direction **S**. As a result of sequential passage of radiation through the absorbing volumetric zones the radiant flux will decrease. Over the segment S the volumetric zone absorbs the following amount of energy

$$\Delta I_l = I_{l-1} \,(1 - \exp\,(-K_l S_l)) \tag{27}$$

with the remaining energy being equal to

$$I_l = I_{l-1} \exp\,(-K_l S_l) \,.$$

Thus, the energy emitted from the volumetric zone l is

$$I_l = I_0 \exp\,(-K_l S_l) \,;$$

the zone $l + 1$ emits

$$I_{l+1} = I_l \exp\,(-K_l S_l) \exp\,(-K_{l+1} S_{l+1}) \,.$$

The following amount of energy will reach the surface zone:

$$I_j = I_0 \prod_{p=l}^{p=l+n} \exp(-K_p S_p) .$$

Using the definition of ψ_{ij}, for the surface zone we will have

$$\psi_{ij} = \frac{1}{NI_0} \sum_{k=1}^{M_{\text{ef}}} \left(I_0 \prod_{p=l}^{p=n} \exp(-K_p S_p) \right)$$

or

$$\psi_{ij} = \frac{1}{M} \sum_{k=1}^{M_{\text{ef}}} \prod_{p=l}^{p=n} \exp(-K_p S_p) . \tag{28}$$

If the irradiated zone j is volumetric, then the fraction of energy that reached this zone is

$$I_{j-1} = I_0 \prod_{p=l}^{p=j-1} \exp(-K_p S_p) .$$

Then the energy absorbed in zone j will be equal to

$$\tilde{I}_j = I_{j-1} (1 - \exp(-K_j S_j))$$

or

$$\tilde{I}_j = I_{01} \prod_{p=l}^{p=j-1} \exp(-K_p S_p) (1 - \exp(-K_j S_j)) .$$

The generalized angular coefficient in the case of zone j being a volumetric one is defined as

$$\psi_{ij} = \frac{1}{M} \sum_{k=1}^{M_{\text{ef}}} \left[\prod_{p=l}^{p=n-1} \exp(-K_p S_p) (1 - \exp(-K_j S_j)) \right] . \tag{29}$$

To realize the given method, we arrange the coordinate axes so as shown in Fig. 3. The equation of the cylinder surface in this coordinate system has the form

$$x^2 + y^2 = r^2 . \tag{30}$$

According to the algorithm proposed by us, one should select randomly the coordinates of emitting points (x_M, y_M, z_M) inside emitting zones. According to [5], the coordinates of the point M on the cylindrical emitting surface in a rectangular coordinate system for section l ($i = l + N$) are the following:

$$x_M = r \cos(2\pi\gamma_\theta) , \quad y_M = r \cos(2\pi\gamma_\theta) , \quad z_M = z_l + \gamma_z \Delta l . \tag{31}$$

In the cylindrical volume the coordinates of the point M for the lth section ($i = l$) are given by the formulas [5]

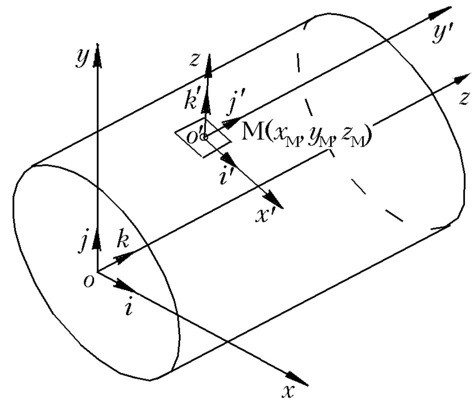

Fig. 3. Toward determination of the direction of a beam for an arbitrary surface.

$$r_M = r\sqrt{\gamma_r}\,,\quad x_M = r_M \cos(2\pi\gamma_\theta)\,,\quad y_M = r_M \sin(2\pi\gamma_\theta)\,,\quad z_M = z_i + \gamma_z \Delta l\,. \tag{32}$$

After the coordinates of the emitting point have been determined, we, according to the algorithm, must select the direction of the beam. In [5] a semi-spherical radiation from the point lying on an arbitrarily oriented surface of second order is considered (see Fig. 3). A mobile coordinate system $o'x'y'z'$ is introduced the $o'z'$ axis of which coincides with the normal at the point M. Expressions for the direction cosines of the beam in the mobile coordinate system have the form

$$\omega_{z'} = \sqrt{1-\gamma_\eta}\,,\quad \omega_{x'} = \sqrt{1-\omega_{z'}^2}\cos(2\pi\gamma_\theta)\,,\quad \omega_{y'} = \sqrt{1-\omega_{z'}^2}\sin(2\pi\gamma_\theta)\,. \tag{33}$$

In order to orient the beam in the original coordinate system $\overline{\overline{oxyz}}$ it is necessary to recalculate the direction cosines given by Eqs. (33) by the transformation formulas

$$\begin{aligned}
\omega_x &= \omega_{x'} \cos\left(\widehat{i'i}\right) + \omega_{y'} \cos\left(\widehat{j'i}\right) + \omega_{z'} \cos\left(\widehat{k'i}\right),\\
\omega_y &= \omega_{x'} \cos\left(\widehat{i'j}\right) + \omega_{y'} \cos\left(\widehat{j'j}\right) + \omega_{z'} \cos\left(\widehat{k'j}\right),\\
\omega_z &= \omega_{x'} \cos\left(\widehat{i'k}\right) + \omega_{y'} \cos\left(\widehat{j'k}\right) + \omega_{z'} \cos\left(\widehat{k'k}\right)
\end{aligned} \tag{34}$$

or

$$\omega_x = \omega_{x'} s_1 + \omega_{y'} s_2 + \omega_{z'} s_3\,,$$

$$\omega_y = \omega_{x'} m_1 + \omega_{y'} m_2 + \omega_{z'} m_3\,,$$

$$\omega_z = \omega_{x'} h_1 + \omega_{y'} h_2 + \omega_{z'} h_3\,.$$

If Φ is the cylindrical surface, i.e., $\Phi = x^2 + y^2 - r^2 = 0$, it is not difficult to find that s_3, m_3, and h_3 are the direction cosines of the normal relative to the original system $\overline{\overline{oxyz}}$. It is known that the direction cosines of the normal to the surface $\Phi(x, y, z) = 0$ are determined at the point

$$s_3 = \frac{1}{S}\frac{\partial \Phi}{\partial x}\,,\quad m_3 = \frac{1}{S}\frac{\partial \Phi}{\partial y}\,,\quad h_3 = \frac{1}{S}\frac{\partial \Phi}{\partial z}\,,$$

where

$$\frac{\partial\Phi}{\partial x} = 2x\ , \quad \frac{\partial\Phi}{\partial y} = 2y\ , \quad \frac{\partial\Phi}{\partial z} = 0\ , \quad S = \sqrt{\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2}\ .$$

For the cylindrical surface at the point M (x_{M}, y_{M}, z_{M})

$$s_3 = \frac{2x_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\ , \quad m_3 = \frac{2y_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\ , \quad h_3 = 0\ .$$

Since the $\overline{\overline{o'y'}}$ axis is parallel to the axis $\overline{\overline{oz}}$, then

$$s_2 = \cos\left(\widehat{j'i}\right) = 0\ , \quad m_2 = \cos\left(\widehat{j'j}\right) = 0\ , \quad h_2 = \cos\left(\widehat{j'k}\right) = 1\ .$$

It is also known that for each line we may write the equality of the form

$$h_1^2 + h_2^2 + h_3^2 = 1\ .$$

For any pair of lines we may write

$$m_1 h_1 + m_2 h_2 + m_3 h_3 = 0\ ,$$

for any combination of three lines the following equality is valid:

$$m_2 h_3 - m_3 h_2 - s_1 = 0\ ,$$

then

$$h_1 = 0\ , \quad m_1 = \frac{2x_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\ , \quad s_1 = -\frac{2y_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\ .$$

Taking into account the fact that radiation occurs in the inner space of the furnace, i.e., by changing the direction of the beam to the opposite, for the direction cosines of the emitting point (x_{M}, y_{M}, z_{M}) of the surface zone of lining we finally obtain

$$\omega_x = \frac{2y_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\,\omega_{x'} = \frac{2x_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\,\omega_{z'}\ ,$$

$$\omega_y = -\frac{2x_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\,\omega_{x'} = \frac{2y_{\mathrm{M}}}{\sqrt{4x_{\mathrm{M}}^2 + 4y_{\mathrm{M}}^2}}\,\omega_{z'}\ , \tag{35}$$

$$\omega_z = -\,\omega_{y'}\ .$$

If there is spherical radiation of a point in the volume, then, according to [5], for the direction cosines we have

$$\omega_z = 1 - 2\gamma_\varphi\ , \quad \omega_x = \sqrt{1 - \omega_z^2}\ \cos\left(2\pi\gamma_\theta\right)\ , \quad \omega_y = \sqrt{1 - \omega_z^2}\ \sin\left(2\pi\gamma_\theta\right)\ . \tag{36}$$

The next stage of the Monte Carlo algorithm is the determination of the length of the segment S from the point to the bounding surface. If given are the point M (x_{M}, y_{M}, z_{M}) and the direction determined by the cosines ω_z, ω_x, and ω_y, then the equation of the straight line will be written in the form

TABLE 1. Distribution of the Temperature of the Gas Volume and Lining Surface along the Furnace Length at Different Diameters and Number of Sections N = 20 and 40

l, m		T, K											
		D = 1 m				D = 2 m				D = 5 m			
N = 20	N = 40	gas		lining		gas		lining		gas		lining	
5	2.5 5	1197	577 1187	1274	623 1262	1247	616 1248	1281	861 1291	1311	778 1406	1136	956 1190
10	7.5 10	1962	1740 1968	1940	1736 1954	1954	1778 1976	1717	1594 1749	1819	1784 1888	1415	1374 1463
15	12.5 15	2111	2074 2117	2096	2061 2110	2076	2067 2098	1836	1829 1863	1830	1903 1874	1462	1496 1495
20	17.5 20	2126	2135 2131	2120	2128 2127	2072	2106 2093	1850	1875 1870	1752	1835 1786	1430	1480 1454
25	22.5 25	2117	2126 2122	2115	2123 2120	2049	2081 2069	1836	1862 1854	1665	1736 1693	1379	1425 1397
30	27.5 30	2109	2118 2113	2106	2116 2111	2027	2058 2046	1819	1843 1834	1600	1655 1622	1334	1372 1349
35	32.5 35	2100	2109 2105	2098	2108 2103	2005	2035 2024	1800	1825 1816	1543	1591 1562	1294	1327 1307
40	37.5 40	2092	2101 2096	2090	2098 2096	1984	2013 2003	1782	1807 1798	1491	1534 1507	1256	1287 1267
45	42.5 45	2083	2092 2088	2081	2090 2085	1963	1992 1981	1765	1789 1780	1444	1482 1457	1221	1249 1231
50	47.5 50	2075	2084 2080	2073	2082 2078	1943	1971 1960	1747	1771 1762	1400	1434 1411	1190	1214 1197
55	52.5 55	2067	2075 2071	2065	2073 2069	1922	1950 1939	1730	1753 1745	1360	1390 1369	1160	1182 1166
60	57.5 60	2058	2067 2063	2056	2065 2061	1902	1929 1919	1713	1736 1729	1322	1349 1329	1132	1151 1137
65	62.5 65	2050	2059 2055	2049	2057 2053	1883	1909 1899	1698	1720 1711	1288	1311 1293	1105	1123 1109
70	67.5 70	2042	2051 2047	2041	2050 2045	1864	1889 1880	1682	1703 1695	1255	1276 1259	1081	1097 1084
75	72.5 75	2034	2042 2038	2032	2041 2037	1845	1870 1861	1666	1687 1679	1224	1243 1228	1057	1072 1060
80	77.5 80	2026	2034 2030	2025	2033 2029	1826	1851 1842	1650	1672 1663	1194	1212 1197	1034	1048 1036
85	82.5 85	2018	2026 2022	2016	2025 2022	1808	1832 1823	1634	1655 1647	1165	1182 1167	1011	1025 1013
90	87.5 90	2010	2018 2014	2008	2018 2012	1787	1814 1804	1613	1639 1629	1133	1152 1136	985	1001 987
95	92.5 95	2000	2010 2006	1994	2009 2002	1753	1793 1780	1576	1617 1599	1094	1118 1097	952	972 953
100	97.5 100	1976	2000 1984	1928	1986 1911	1673	1754 1698	1469	1562 1456	1031	1070 1030	887	928 874

TABLE 2. Distribution of the Relative Value of the Longitudinal Radiation Flux ΔQ_{long} along the Furnace Length (the value is constant over an interval) at $D = 5$ m and Number of Sections $N = 20$ and $N = 40$

l, m		ΔQ_{long}, %			
$N = 20$	$N = 40$	$N = 20$		$N = 40$	
		gas	lining	gas	lining
5	2.5 5	54.8	46.0	81.4 67.9	79.5 55.2
10	7.5 10	36.1	23.0	57.4 55.3	41.0 39.6
15	12.5 15	38.1	25.4	55.5 56.5	39.9 40.9
20	17.5 20	40.7	27.4	57.1 57.9	41.2 41.7
25	22.5 25	42.1	28.2	58.3 58.4	42.0 42.0
30	27.5 30	41.9	27.8	58.2 58.0	41.8 41.5
35	32.5 35	41.3	27.4	57.7 57.4	41.3 41.1
40	37.5 40	40.7	26.9	57.0 56.7	40.8 40.5
45	42.5 45	40.0	26.5	56.3 56.1	40.3 40.1
50	47.5 50	39.4	26.1	55.7 55.5	39.9 39.7
55	52.5 55	38.9	25.7	55.2 54.9	39.4 39.3
60	57.5 60	38.4	25.3	54.7 54.4	39.1 38.9
65	62.5 65	37.9	25.0	54.2 53.9	38.7 38.6
70	67.5 70	37.4	24.7	53.7 53.5	38.4 38.3
75	72.5 75	37.0	24.4	53.2 53.0	38.1 38.0
80	77.5 80	36.5	24.1	52.8 52.5	37.8 37.7
85	82.5 85	35.8	23.7	52.3 52.0	37.5 37.3
90	87.5 90	34.8	23.0	51.6 51.0	37.1 36.7
95	92.5 95	32.4	21.7	50.2 48.8	36.2 35.5
100	97.5 100	24.7	15.6	46.0 37.9	33.9 25.6

$$x = x_M + \omega_x S\,, \quad y = y_M + \omega_y S\,, \quad z = z_M + \omega_z S\,.$$

If the bounding surface is a plane that divides the volumetric zones and is described by the equation $z = z_j$, then the distance to the plane is given by

$$S = \frac{z_j - z_M}{\omega_z}\,. \tag{37}$$

The length of the segment S to the cylindrical surface is determined by solving the quadratic equation

$$(x_M + \omega_x S)^2 + (y_M + \omega_y S)^2 - r^2 = 0\,, \tag{38}$$

$$S = -\frac{(2x_M\omega_x + 2y_M) + \sqrt{(2x_M\omega_x + 2y_M)^2 - 4\,(\omega_x^2 + \omega_y^2)\,(x_M^2 + y_M^2 - r^2)}}{2\,(\omega_x^2 + \omega_y^2)}\,.$$

Results. Based on the constructed mathematical model (1)–(38), an investigation of the influence of longitudinal radiation in high-temperature thermal processes in cylindrical furnaces was carried out. The numerical investigations were conducted with the following initial data: diameter of the furnace D = 1, 3, and 5 m, furnace length 100 m, temperature of the natural gas supplied for combustion 293 K, temperature of supplied air 286 K, fuel flow rate B = 0.8 m^3/sec, concentration of oxygen in fuel gases 1.4%, and emissivity of the lining $\varepsilon_{lin} = 0.9$.

Table 1 presents the distribution of the temperature of the gas and lining along the length of the furnace depending on its diameter. The furnace was divided into 20 and 40 sections. As is seen, with increase in the furnace diameter the difference between the temperatures of the gas and lining in a certain section of the furnace increases.

Table 2 presents the values of the longitudinal component ΔQ_{long} for each zone of the furnace sections. The furnace of diameter 5 m was divided into 20 sections. An analysis of the results shows that for large diameters the fraction of the longitudinal component is substantial. Especially large values of longitudinal radiation are observed in the cold zone of flame combustion. For D = 5 m the average value of the longitudinal radiation for the volumetric zone of the furnace is equal to 36% and the surface one 24%. Correspondingly, with an increasing number of sections the components of longitudinal radiation increase. Table 2 presents also the distribution of the fraction of longitudinal radiation for a furnace divided into 40 zones, where the average value of longitudinal radiation is equal to 58% for the volumetric zone and 43% for the surface one.

Thus, during numerical simulation of high-temperature processes the role of the longitudinal radiation is substantial. Consequently, the assumption on the exclusively radial propagation of the radiant flux may lead to considerable errors. In such cases, the zone method with the use of the Monte Carlo method of statistical tests may be efficient for modeling radiant heat transfer.

The reliability of the results obtained is confirmed by experimental investigations on industrial rotating furnaces used for roasting fluorine-free phosphates at the Uvarovka Chemical Plant of Tambov district [17].

The proposed technique of calculation can be used in designing and optimizing the thermal regimes of operation of rotating roasting furnaces in chemical and metallurgical industry.

NOTATION

a, b, empirical coefficients; A, coefficient in the equation of radiative heat transfer; B, flow rate of the fuel (natural gas) supplied to the atomizer, m^3/sec; c, heat capacity, J/(m^3·K); D, diameter of the furnace, m; f, resolving angular coefficient of radiation; F, surface area, m^2; G, volumetric flow rate, m^3/sec; I, energy of a packet of photons; K, coefficient of absorption, 1/m; l, distance from the outlet section of the nozzle to the section considered, m; Δl, length of a section, m; L, flow rate of air spent on combustion of 1 kg of fuel, m^3/kg; m, h, and s, direction cosine of mobile coordinate system $\overline{o'x'y'z'}$; M, number of tests; n, number of volumetric zones that are crossed by the beam **S** until its meeting with the bounding surface of zone j; N, number of the sections into which the furnace is divided;

Nu, Nusselt number; P, total content of H_2O and CO_2 in combustion products, %; q, specific heat of combustion of natural gas, J/m^3; Q, heat flux, W; ΔQ, resultant heat flux due to the heat transfer with the moving medium, W; r, inner radius of the furnace, m; R, coefficient of reflection; Re, Reynolds number; S, length of the beam segment, m; t and T, temperature, ^{o}C and K; U, volume of section, m^3; V, gas volume on burning of fuel, m^3; w, average velocity of gas motion in a rotating furnace, m/sec; x, y, z, coordinates of the emitting point; α, average coefficient of heat transfer, W/(m^2·K); β, integral inflow of air into a flame, W/(m^2·K); γ_r, γ_θ, γ_η, γ_z, and γ_φ, random numbers uniformly distributed in the interval [0, 1]; ε, emissivity; ϑ, coefficient of the excess of air supplied for combustion; λ, thermal conductivity of gas, W/(m·K); ν, coefficient of kinematic gas viscosity, m^2/sec; ρ, density of a gas mixture, kg/m^3; σ, coefficient of radiation of a blackbody, W/(m^2·K^4); χ, degree of fuel burning out; Φ, cylindrical surface; ψ, generalized angular coefficient of radiation; ω_z, ω_x, ω_y, values of direction cosined at the point (x, y, z) in the original coordinate system. Subscripts and superscripts: i, j, k, p, nomber of a zone; l, nomber of the section considered; α, real value; Σ, overall; g, gas; r, radiant; inf, inflow; inc, incident; long, longitudinal; s, sooty particles; com, combustion; int, intrinsic; th, theoretical value; lin, lining; f, flame; ef, effective.

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